Coupled Flow and Geomechanics Modeling for Fractured Poroelastic Reservoirs

by

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Abstract

The design and evaluation of hydraulic fracture modeling is critical for efficient production from tight gas and shale plays. The efficiency of fracturing jobs depends on the interaction between hydraulic (induced) and naturally occurring discrete fractures. We describe a coupled reservoir-fracture flow model which accounts for varying reservoir geometries and complexities including non-planar fractures. We utilize different flow models such as Darcy flow and Reynold’s lubrication equation for fractures and reservoir to closely capture the physics. Furthermore, the geomechanics effects have been included by considering a multiphase Biot’s model. An accurate modeling of solid deformations necessitates a better estimation of fluid pressure inside fracture. We model the fractures and reservoirs explicitly, which allows us to capture the flow details and impact of fractures more accurately. The approach presented here is in contrast with existing averaging approaches such as dual and discrete-dual porosity models where the effects of fractures are averaged out. A fracture connected to an injection well shows significant width variations as compared to natural fractures where these changes are negligible. Furthermore, the capillary pressure contrast between the two is accounted for by utilizing different capillary pressure curves. We present several numerical tests, including a field scale case study, to illustrate the above features and their impact on recovery predictions.

Keywords. phase-field; pressurized fractures; iterative coupling algorithm; reservoir simulation

Introduction

Multiphase flows in fractured reservoirs are of immense importance for energy security. The recovery of hydrocarbons in a reservoir is strongly dependent on the fractures, both natural and artificial. To model these processes, it is imperative to have a reliable model that captures the effects of these fractures with high accuracy. Moreover, the geometric complexities of the fractures require sophisticated numerical approaches. Moreover, the pore pressure from the flow may also cause geomechanical effects. These effects are more pronounced when the fractures are present. The geomechanical effects and multiphase flows in a fractured porous medium are modeled by coupled nonlinear system of differential equations. Additionally, we need to account properly for reservoir heterogeneities due to discontinuous rock properties. This manifests in the form of discontinuous changes in
absolute permeability, relative permeability and capillary pressure curves. The simulation of this system presents important challenges from both modeling and computational points of view. In this work, we perform a quantitative and qualitative study of these effects and describe a numerical approach for solving this problem. Our approach offers high accuracy and fidelity in capturing the physics of the problem.

The model consists of two parts: geomechanics and flow equations. The flow equations in a fractured reservoirs have been a subject of intensive study by several authors. Our approach fully resolves the flow by considering separate equations for the fractures and reservoirs which are coupled together. As we shall see later in this section (see Fig 1), this resolution of fracture flow is important as a crude approximation may lead to unacceptable errors. The fracture flow model is formulated by reducing a higher dimensional ($\mathbb{R}^d$) model to a lower dimensional ($\mathbb{R}^{d-1}$) manifold by averaging procedure. The derivation of this reduction procedure has been undertaken by Martin et al. (2005) and Frih et al. (2008) for Darcy and Forchheimer flows. The reduction leads to a set of equations defined for both fractures and reservoirs with fractures acting as interfaces inside the reservoirs. The purview of these studies is however limited to single phase flow accounting for permeability heterogeneity at the reservoir-fracture interface. We will follow a similar approach here for multiphase flows.

We begin by reviewing some of the work that is relevant to our presented approach. Hoteit and Firoozabadi (2008a) present a good introduction of different finite volume and finite element based discretizations for such problems. A mixed finite element for multi-phase, reservoir-fracture flow model was proposed by Hoteit and Firoozabadi (2005, 2006) assuming a cross-flow equilibrium across reservoir-fracture interface. This assumption was later removed (Hoteit and Firoozabadi (2008b)) by considering additional degrees of freedom at the interface. Here, they consider a hybrid mixed method to solve an implicit pressure equation along with a higher order discontinuous Galerkin method with a slope limiter for an explicit saturation update following an IMPES (implicit pressure explicit saturation) scheme. Further, Grillo et al. (2010) discuss density driven flows where fractures are represented as 2 and 3-dimensional manifolds assuming a multi-component, single-phase flow system. A finite-volume based numerical discretization is used, with each fracture having two degrees of freedom. Our method is inspired by Hoteit and Firoozabadi (2008a) and uses mixed finite element methods and an IMPES solution scheme. We differ in the usage of a multipoint flux scheme based upon an appropriate choice of finite element spaces and quadrature rule (Ingram et al. (2010), Wheeler and Yotov (2006), Arbogast et al. (1997), Wheeler et al. (2011a)). This approach provides flexibilities in capturing the complex geometric features of fractures.

Earlier, a finite-volume based approach was presented by Bastian et al. (2000) and Monteagudo and Firoozabadi (2004) on unstructured grids. The reservoir-fracture interfaces have only one pressure and saturation degree of freedom and thus jumps in these quantities cannot be considered, thereby assuming a cross-flow equilibrium. Their model takes into account capillary pressure discontinuity due to rock heterogeneity. However, saturation calculation at the interface requires inversion of capillary pressure, which may pose problems if the capillary pressure curve is either identically zero or has a small gradient. Monteagudo and Firoozabadi (2007) address this issue by using two different formulations based upon threshold values requiring calibration. More recently, a coupled flow in a fractured-reservoir
models is considered in Al-Hinai et al. (2013) where different discretization schemes were utilized inside the fracture (mimetic finite difference) and reservoir (mixed finite element) in the absence of capillary pressure and time invariant fracture permeabilities.

The geomechanical effects in the reservoir are accounted for by considering an elastic deformable medium. The theoretical groundwork for one-dimensional flow in a deformable porous medium was first developed by Terzaghi (1943). This was later extended to a general theory of three-dimensional consolidation for anisotropic and heterogeneous materials by Biot in his subsequent works (Biot (1941a,b, 1955)). For the coupling of multiphase flow and geomechanics, Settari and Walters (2001) categorize the various schemes as decoupled and explicit, iterative and fully coupled. An explicit or loose coupling has lower computational time with little control on solution accuracy. Further, a time-step size guidance is often required for the geomechanics solve, which is empirical or heuristic in nature (Dean et al. (2006)). On the other hand, a fully implicit method is both accurate and stable, however solving the implicit system of equations results in a complex nonlinear system requiring suitable linearization schemes and specialized linear solvers for convergence. In this work, we apply an iterative coupling scheme based on fixed stress splitting for which convergence analysis has been done in Mikelic and Wheeler (2012); Mikelic et al. (2014); Kim et al. (2012). The analysis shows that the iterative scheme converges geometrically. An iterative coupling approach combines the advantages of both methods while maintaining numerical solution accuracy, fast convergence, and ease of implementation in existing legacy flow simulators.

The poroelastic models rely upon pore-pressure to account for changes in material stress. Accuracy of flow models, especially in capturing sharp pressure changes across a reservoir-fracture interface, plays a pivotal role when poroelastic behavior of reservoir and fracture mechanics comes into play. For a single phase flow, Ganis et al. (2013) presents a coupled flow model fractures in a poroelastic medium. The multipoint flux mixed finite element method (MFMFE) used in this work is defined for general hexahedral grids with non-planar edges. This allows non-planar fracture geometry to be captured. A detailed numerical analysis for single phase reservoir-fracture flow coupling presented here can be found in Girault et al. (2013).

There are several novelties in this work. We use hexahedral grids with an MFMFE scheme which allows non-planar fractures and an accurate computation of a locally mass-conservative flow profile. Secondly, we resolve the flow equations for both the fractures and reservoir in a coupled manner. This is achieved by assuming a lubrication equation inside the fractures and multiphase Darcy law for the reservoir. Thirdly, the fixed stress splitting scheme for the geomechanics effects in a reservoir has been extended to include fractures where the permeabilities of the fracture are functions of the deformations. Furthermore, our numerical results have been compared with physical core-scale experiments and benchmark problems demonstrating the capabilities of our approach.

We begin with a simplified 2D example to qualitatively outline some of the differences between conventional modeling techniques and the approach presented here. This is followed by a reservoir-fracture flow and geomechanics model formulation with a brief description of conservation and constitutive equations along with the required closure conditions. We then discuss a choice of boundary, interface and initial conditions used to describe the problem. Next in the algorithm and discretization section we briefly outline the spatial and temporal
An illustrative example

In this section, we motivate this work by emphasizing the need to fully resolve the fracture geometry. The following simplified example underlines the need for a detailed modeling. This will be achieved by considering and comparing different approaches for fractured reservoir flow modeling. A reservoir domain of size 10 ft × 10 ft is considered with bottom-hole pressure specified injection (520 psi) and production (500 psi) wells located at diagonally opposite ends. Further, a homogeneous and isotropic reservoir permeability of 5 mD and porosity of 0.2 are assumed. Fig. 1 shows saturation profiles for three approaches: (1) an average permeability representation (1st row, 1st column), where the dotted red region has been assigned higher average permeability of $5 \times 10^6$ mD assuming the fracture goes through those blocks, (2) a meshed-in representation where the fracture itself is gridded (1st row, 2nd column) and has a permeability value of $5 \times 10^{10}$ mD, and finally, (3) the interface approach presented in this work (1st row, 3rd column) where the fracture is represented as a lower dimensional surface shown by the red dotted line. The capillary pressure is taken to be identically zero everywhere. The relative permeability curves are shown in Fig. 2.

Note that an actual fracture width of 1 mm is used for the interface approach when compared to the meshed-in approach where the grid-block width normal to the fracture surface is 1cm. The saturation profiles at $t = 50$ and 100 days (2nd and 3rd row in Fig. 1) show differences in sweep pattern for the different approaches. The averaging approach mobilizes additional fluid resulting in overestimation of recoveries. The meshed-in approach, although more accurate, poses a few challenges. A mesh refinement is required to capture the fracture, which may not always be possible. Furthermore, a time-step size restriction due to an order of magnitude difference between fracture and reservoir grid block sizes is observed. A relatively large fracture width (1cm) has been chosen due to time-step size restriction imposed by CFL (Courant-Friedrichs-Lewy) condition. The interface approach overcomes these issues while preserving the physics. Fig. 1 (2nd row 3rd column) shows fluid entering fracture at one end and leaving at the other end without mobilizing additional fluid in between. In the numerical results section, we further elaborate on the merits and the limitations of the averaging approach. We also show that the orientation and location of fractures are important parameters in determining the choice of fracture modeling.

Model Formulation

In this section, we provide a brief description of the fractured reservoir flow and geomechanics model where fractures are treated as lower dimensional manifolds in $(\mathbb{R}^{d-1})$ in a reservoir domain $\Omega \in \mathbb{R}^d$, ($d = 2$ or 3). The model has two components: two-phase flow and mechanics. The modeling equations are defined separately in the reservoir and on the fracture surfaces along with the associated interface conditions. For the flow model, a slightly compressible, two-phase, locally mass conservative Darcy flow is assumed for the
reservoir domain and a lubrication equation for the fractures. We further assume oil and water are the two phases denoted by subscripts $o$ and $w$, respectively. A schematic of a fractured reservoir is shown in Fig. 3. Note that the fracture geometry is not necessarily planar and as explained later, our numerical method allows for non-planar geometries.

We treat the fracture as a pressure specified internal boundary in the reservoir domain and provide the jump in flux across this interface as a leakage term to the fracture. The fracture pressure is also treated as an internal traction boundary condition for the reservoir geomechanics. The resulting jump in displacements (fracture width or aperture) is used to calculate fracture permeability. For the model description, we consider a fractured reservoir domain $\Omega$ where the fracture is represented as an interface ($\Gamma$) with two surfaces ($\Gamma^\pm$), as shown in Fig. 3. Here $\partial \Omega^N$ and $\partial \Omega^D$ represent the Neumann and Dirichlet parts, respectively of the external boundary of the reservoir domain $\Omega$. 

Figure 1: Saturation profiles for averaging, meshed-in and interface approaches (left to right) for time $t = 0, 50$ and 100 days (top to bottom).
Equations in the reservoir \((\Omega \backslash \Gamma)\) We begin by describing the flow equations everywhere except for the fracture interface \(\Gamma\).

**Flow equations** The mass-conservation equation for the phase \(\beta\) reads,

\[
\frac{\partial}{\partial t} (\phi^* S_\beta \rho_\beta) + \nabla \cdot z_\beta = q_\beta. \tag{1}
\]

Here, \(\phi^*\) is the fluid fraction, \(S_\beta\) the saturation, \(\rho_\beta\) the density, \(z_\beta\) the flux of phase \(\beta = o\) (oil phase), \(w\) (water phase). The source/sink term \(q_\beta\) is treated using an appropriate well model (Peaceman (1978)). The Darcy equation relates the flux \(z_\beta\) to the gradient of the phase pressure and is given by

\[
z_\beta = -K \rho_\beta \frac{k_{r\beta}}{\nu_\beta} (\nabla p_\beta - \rho_\beta g). \tag{2}
\]

In the above, \(K\) is a full tensor absolute permeability, \(k_{r\beta}\) the relative permeability, and \(\nu_\beta\) the viscosity of phase \(\beta\). The second term in the parenthesis models the effect of gravity.

**Mechanics** The displacement \(u\) of the porous medium is described by the quasistatic poroelastic equations. Eqn. (3) represents the momentum conservation (force balance) and Eqn. (4) the constitutive equation relating stress \((\sigma^{\text{por}})\) and displacements \((u)\):

\[
-\nabla \cdot \sigma^{\text{por}}(u, p_{\text{rel}}) = f, \tag{3a}
\]
\[ f = \left[ \rho_s(1 - \phi^*) + \phi^* \sum_{\beta} \rho_{\beta} S_{\beta} \right] g, \quad (3b) \]
\[ \phi^* = \phi_o + \alpha \nabla \cdot \mathbf{u} + \frac{1}{M_{p_{\text{ref}}}}. \quad (3c) \]

\[ \sigma^{\text{por}}(\mathbf{u}, p_{\text{ref}}) = \sigma(\mathbf{u}) - \alpha p_{\text{ref}} \mathbf{I}, \quad (4a) \]
\[ \sigma(\mathbf{u}) = \lambda (\nabla \cdot \mathbf{u}) \mathbf{I} + 2 \mu \mathbf{\epsilon}(\mathbf{u}), \quad (4b) \]
\[ \mathbf{\epsilon}(\mathbf{u}) = \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^T). \quad (4c) \]

Here, \( \sigma^{\text{por}} \) is the Cauchy stress tensor, \( \mathbf{\epsilon} \) the strain tensor, \( f \) the body force, \( \rho_s \) density of the solid matrix, \( g \) acceleration due to gravity, \( \alpha \) the Biot coefficient, \( p_{\text{ref}} \) the reference phase pressure, \( \mathbf{I} \) the identity matrix, \( \phi_o \) the reference porosity, \( \mu \) and \( \lambda \) are Lamé parameters.

**Equations in the fracture** (\( \Gamma \)) A lubrication equation is assumed as the constitutive relation between fracture fluxes (\( z^\Gamma_\beta \)) and gradient of fracture pressure (\( p^\Gamma \)). Here, the fracture gradient (\( \nabla \)) and divergence (\( \nabla \cdot \)) operators are defined on a lower dimensional space (\( \mathbb{R}^{d-1} \)). **Eqns.** (5) and (6) represent the mass conservation and Darcy’s law for the phase ‘\( \beta \)’ in the fracture domain:

\[ \frac{\partial}{\partial t} (w S^\Gamma_\beta \rho^\Gamma_\beta) + w \nabla \cdot z^\Gamma_\beta = q^\Gamma_{\beta} + q_{\text{leak}}, \quad (5) \]
\[ z^\Gamma_\beta = -K^\Gamma_{\beta} \rho^\Gamma_{\beta} \frac{k_{r\beta}}{\nu_{\beta}} (\nabla p^\Gamma_\beta - \rho^\Gamma_{\beta} g). \quad (6) \]

Here, \( q_{\text{leak}} \) is the fracture leakage term as defined below. The absolute permeability \( K^\Gamma \) is given by **Eqn.** (7) where \( (w) \) is the fracture width,

\[ K^\Gamma = \frac{w^2}{12}. \quad (7) \]

**Closure conditions** A slightly compressible, equation of state relating fluid densities and pressures is assumed (**Eqn.** (8)) for both reservoir and fracture. Further, the capillary pressure and saturation constraints are given by **Eqns.** (9) and (10), respectively. For simplicity of notation we let \( \star = \Omega \setminus \Gamma \) or \( \Gamma \) and for a given function \( f^{\Omega \setminus \Gamma} \) to be equal to \( f \). Thus, we have

\[ \rho^*_{\beta} = \rho_{\beta 0} \left[ 1 + c_{\beta} (p^*_{\text{ref}} + p^*_{c_{\beta}} - p_0) \right], \quad (8) \]
\[ p^*_{c_{\beta}} (S^*_{\beta}) = p^*_0 - p^*_w, \quad (9) \]
\[ S^*_{w} + S^*_{0} = 1. \quad (10) \]

Here, \( c_{\beta} \) is the fluid compressibility, \( p_{c_{\beta}} \) the capillary pressure for the fluid phase \( \beta \) and \( S_{\text{ref}} \) the reference phase saturation. The relative permeabilities are continuous functions of reference phase saturation (\( S_{\text{ref}} \)) for both reservoir and fracture. A more general table based capillary pressure and relative permeability curve description has also been implemented.
**Boundary, interface and initial conditions** For the sake of simplicity, we consider no-flow or pressure specified external boundary conditions for the reservoir domain ($\Omega$):

$$z_\beta \cdot n = 0 \text{ on } \partial \Omega^N. \quad (11)$$

For saturations, we specify Dirichlet boundary conditions on $\partial \Omega^D$. However, the choice is not restrictive and is used for convenience. That is,

$$p_{\text{ref}} = p^D \text{ on } \partial \Omega^D,$$

$$S_{\text{ref}} = S^D \text{ on } \partial \Omega^D. \quad (12)$$

Furthermore, the proposed model assumes a pressure-specified internal boundary condition (Eqn. (13)) given by,

$$p_{\text{ref}} = p^D \text{ on } \Gamma^\pm. \quad (13)$$

Assuming capillary pressure functions are monotone functions whenever uniformly non-zero and therefore invertible, three modeling choices are considered.

**Case I** $p_c$ is identically equal to zero everywhere:

$$S_{\text{ref}} = S^D \text{ on } \Gamma^\pm. \quad (14)$$

**Case II** $p_c$ is strictly greater than zero everywhere:

$$S_{\text{ref}}^- = (p_{c,\text{ref}}^-)^{-1}(p_{c,\text{ref}}^\Gamma(S_{\text{ref}}^\Gamma)) \text{ on } \Gamma^-, \quad S_{\text{ref}}^+ = (p_{c,\text{ref}}^+)^{-1}(p_{c,\text{ref}}^\Gamma(S_{\text{ref}}^\Gamma)) \text{ on } \Gamma^+. \quad (15)$$

**Case III** $p_c$ is strictly greater than zero in the reservoir and identically equal to zero in the fracture:

$$S_{\text{ref}}^- = (p_{c,\text{ref}}^-)^{-1}(0) \text{ on } \Gamma^-, \quad S_{\text{ref}}^+ = (p_{c,\text{ref}}^+)^{-1}(0) \text{ on } \Gamma^+. \quad (16)$$

Here $(p_{c,\text{ref}}^\pm)^{-1}$ is the inverse of capillary pressure on top and bottom (or left and right) surfaces of the fracture. Note that fluid mass exchange between reservoir and fracture is accounted for by the leakage term ‘$q_{\beta}$’ in Eqn. (5). Assuming the two fracture surfaces have the same normal, a jump in reservoir fluxes and displacements across the fracture interface ($\Gamma^\pm$) is provided as a leakage (source term) and fracture width, respectively to the fracture mass conservation:

$$q_{\beta} = [z_\beta \cdot n]^\Gamma = z_\beta \cdot n|_{\Gamma^-} - z_\beta \cdot n|_{\Gamma^+}, \quad (17a)$$

$$w = [u \cdot n]^\Gamma = u \cdot n|_{\Gamma^-} - u \cdot n|_{\Gamma^+}. \quad (17b)$$

The geomechanics boundary conditions are given by

$$(\sigma_{\text{por}}(u, p)n)|_{\Gamma^\pm} = -p_{\text{ref}}^\Gamma n, \quad (18a)$$
where $\partial\Omega_s$ are the external boundaries. The initial conditions at time $t = 0$ are as follows:

\begin{align*}
    p_{\text{ref}} &= p_0, \quad (19) \\
    S_{\text{ref}} &= S_0, \quad (20) \\
    u &= u^0, \quad (21)
\end{align*}

**Algorithm and discretization** A fixed stress iterative coupling scheme, Settari and Walters (2001); Mikelić and Wheeler (2012), is employed as shown in Fig. 4a. Here we iterate between the flow solution assuming a fixed stress field and the mechanics solution assuming fixed pressure and saturation fields. For the reservoir geomechanics equations, a continuous Galerkin (CG) finite element method is used for spatial discretization. The mechanics solve provides a jump in displacements across the fracture interface (fracture width) which is used to calculate the fracture permeability (Eqn. 7). The fracture pressure from the reservoir-fracture flow solve is then treated as a traction boundary condition for the mechanics solve. Iterations are performed until a desired tolerance $\epsilon_1$ is achieved. Please note that the fracture widths vary both spatially and temporally. Our formulation has been extended to propagating fracture based on a phase field approach Wick et al. (2013); Mikelić et al. (2014); Wheeler et al. (2014)

The MFMFE method, developed by Wheeler and Yotov (2006) for general hexahedra, is used for spatial discretization of reservoir and fracture flow equations. Mixed finite element methods are preferred over other variational formulations due to their local mass conservation and improved flux approximation properties which includes diagonal flow across a
grid-block. An appropriate choice of mixed finite element spaces and degrees of freedom based upon the quadrature rule for numerical integration (Wheeler et al. (2011b); Wheeler and Xue (2011)) allow flux degrees of freedoms to be defined in terms of cell-centered grid-block pressures adjacent to the vertex. A 9 and 27 point pressure stencil is formed for logically rectangular 2D and 3D grids, respectively.

The flow equations are solved using an iteratively-coupled, implicit pressure explicit saturation (IC-IMPES) scheme as shown in Fig. 4b. The reference phase pressure is solved implicitly by solving the total mass conservation equation with a backward Euler time discretization assuming the reference phase saturations are given. This is followed by an explicit update of reference phase saturations using a forward Euler time discretization for the phase mass conservation equation. The solution algorithm allows for smaller saturation time-step sizes than pressure time-steps. Further, the Courant-Frederichs-Lewy (CFL) condition on the explicit saturation updates are then obeyed using different time-step sizes for the reservoir and the fracture. The fluid property data for intermediate saturation time steps are calculated by linear interpolation of pressure. The demarcation of reservoir and fracture as separate domains allows for special treatment of computationally challenging regions. Please note that since the pressures are solved implicitly, there are no restrictions on the pressure time-step size. A tolerance of \( \epsilon_2 \) determines convergence of the iterative scheme.

We finally turn our attention to the non-linear, reservoir-fracture flow system. The reservoir pressure solve provides a jump in fluxes across the fracture interface. These in turn act as source/sink terms for the fracture pressure solve. The resulting fracture pressure is then treated as a pressure specified internal boundary for the reservoir domain. We couple the reservoir and fracture pressure solves by iterating between the two implicit systems until a desired tolerance \( \epsilon_3 \) is reached. Fig. 4c provides an outline of the coupled reservoir-fracture flow model used in this work.

**Results**

In this section, we consider a number of numerical experiments to demonstrate our modeling and computational approaches. We begin with validation of the coupled reservoir-fracture flow model by comparing with physical experimental results for spontaneous imbibition of the wetting phase. The second numerical experiment studies the significance of fracture orientations for recovery processes in a reservoir. An injection scenario for a multi-stage hydraulic fracture is shown in the third example. The fourth example demonstrates stress field reorientations for injection and production from a hydraulic fracture. Finally, a field case for Frio Juntunen and Wheeler (2012)) is presented showing long term production from a fractured reservoir with multiple injection and production wells. The numerical experiments have been conducted for both lab scale as well as field scale. Please note that the fracture aperture (or width) is time invariant and varies spatially from 1 mm - 3 mm along the fracture length in all numerical experiments except example 4. For the couple flow and mechanics the above is used as an initial guess since fracture widths vary spatially and temporally and are solved as a part of the system of equations.

**Capillary imbibition in a fractured core** We compare the results of our numerical model to experimental data, given by Karpyn (2005), for a fractured Berea sandstone core. The core is initially saturated only with water \( (S_w = 1.0) \) followed by a primary
Table 1 provides fluid and rock property information for the core. This example demonstrates that the model can be used to simulate both core-scale and later field-scale scenarios while accurately capturing the physics. The cleaned Berea core is conventionally water wet as can be seen in the matrix relative permeability and capillary pressure curves. Figs. 6 and 7 show experimental saturation profiles obtained using digital radiography and numerical results at different time instances, respectively. Furthermore, the fracture width varies spatially for an accurate depiction of fracture flow. The saturation profiles and average saturations are in good agreement with experimental values.
Discrete natural fractures In the introduction section, we presented a single fracture example to motivate a detailed interface based modeling approach. In the introduction, we presented an example (Fig. 1) where the saturation front channeled through the fracture thereby reducing sweep area. Here, we present a similar case with two discrete fractures in a reservoir domain of size 10 ft × 10 ft (approximately) with bottom-hole pressure specified injection (520 psi) and production (500 psi) wells located at diagonally opposite ends. The reservoir and fluid property data along with initial and boundary conditions remain unchanged.
The fracture closer to the injector (Fig. 8) acts as a shield against the fluid front preventing it from channeling through the other fracture thereby improving sweep area. Thus a fracture can enhance or deteriorate sweep based upon its orientation to the fluid front. This example shows the impact of accurately capturing non-planar fracture geometries and their orientation with respect to the reservoir as well as each other. We also infer that fractures orthogonal to the line joining injector and producer will increase recovery efficiency whereas the one parallel to this line will be detrimental to the recovery of hydrocarbons. It is interesting to note that the former type can be represented using a permeability averaging based approach without significant loss of accuracy. However, the latter still requires a high resolution modeling approach to maintain accuracy. Exploiting the cheaper computational cost of averaging and incorporating this in our detailed modeling will be addressed elsewhere. The insights from this example can be used for studying field scale fractured-reservoirs and as an assistive tool during various planning and developmental stages.

**Multi-stage hydraulic fracture** A three-stage hydraulic fracture with fracture aperture varying along the length is considered. In this example, we stress on the interaction between hydraulic fractures and their consequent impact on injectivity enhancement. Fig. 9 gives a schematic of the problem description. We consider a reservoir domain of size 200 ft × 600 ft × 300 ft with three hydraulic fractures (shaded green) connected to a bottom-hole pressure specified (1000 psi) injection well (shaded blue). A no-flow boundary condition is assumed everywhere except for a part of external boundary (shaded red) where pressure has been specified (400 psi) to show the effect of boundary conditions on injectivity.

Figure 9: Schematic of a three-stage hydraulic fracture connected to a well-bore.

Note that the fracture geometry is non-planar and the depiction in Fig. 9 is a simple representation. The fracture half-lengths are approximately 50 ft with apertures varying from 3mm at the center to 1 mm towards the edges. Table 2 provides the reservoir and fluid property data along with the initial conditions. Fig. 10 shows the pressure (top) and saturation (bottom) at three time instances. The
pressure profile remains almost invariant with time, however saturation profile indicates differences in injectivities from the three fractures. These differences arise due to a combined effect of proximity to other hydraulic fractures and the external boundary conditions. The fracture closest to the pressure specified boundary (least shielded) exhibits maximum injectivity whereas the one farthest (most shielded) contributes the least.

Figure 10: Pressure (top) and saturation (bottom) profiles at $t = 1$, 2 and 6 days (from left to right).

A detailed analysis indicates that an optimal fracture spacing which maximizes injectivity can be achieved while minimizing screening effects for the current setting. It is also seen that decreasing fracture half-lengths as the proximity to pressure boundary increases leads to similar results.

**Coupled flow and mechanics** In this example, we demonstrate the effect of fracture on the stress field similar to stress field reorientation studies presented by Roussel and Sharma (2009). A reservoir domain of size 250 ft $\times$ 250 ft with a single fracture of half-length 25 ft is assumed with fracture apertures varying during injection and production stages from 0.01 ft to 0.05 ft. The Youngs modulus, Poissons ratio and max and minimum stress values are taken to be $7.3 \times 10^6$ psi, 0.2, 6400 and 6300 psi respectively. The initial pressure for the injection and production cases is 500 psi and 5000 psi, respectively. A fracture pressure specification of 5000 psi and 2000 psi was assumed for injection and production cases, respectively with no flow external boundary conditions. We enforce a zero tangential displacement condition at the midpoints of the domain edges to avoid rigid body motion. The reservoir property data is given in Table 2 and is same as in the previous example. Fig. 11 shows a schematic of the problem description.

<table>
<thead>
<tr>
<th>TABLE 2—Reservoir properties, Multistage fractures</th>
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<tbody>
<tr>
<td>$\phi$</td>
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<tr>
<td>$c_w$</td>
</tr>
<tr>
<td>$\rho_w$</td>
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<tr>
<td>$\nu_w$</td>
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<td>$S_o^w$</td>
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Figs. 12 and 13 show variations of principal stress directions and magnitude contours for a single fracture injection and production cases, respectively. It is observed that the principal stress directions around the fracture do not vary significantly for the injection case. However, the changes away from the fracture are primarily due to boundary conditions. Note that we solve on a full domain compared to the quarter domain problem presented by Roussel and Sharma (2009) owing to symmetry arguments. On the other hand, a stress field re-orientation occurs around the fracture very early for the production case. This is strongly influenced by the difference between initial pressure and bottom-hole pressure.

**Frio field case with natural fractures** In this example, we show an extension to field scale fractured reservoirs. **Fig. 14** shows a section of Frio field (Juntunen and Wheeler
Figure 13: Stress magnitude (contour, psi) and principal stress direction (vector) for a single fracture production case for $t = 0.02, 0.1, 6.0$ days (top row, left to right) and $t = 2.5, 7.5$ and 20 days (bottom row, left to right).

(2012)) with discrete natural fractures (shaded orange) with 9 pressure-specified wells: 6 injectors (4000 psi) and 3 producers (2000 psi). The MFMFE discretization allows for accurate representation of reservoir as well as fracture geometries. The reservoir dimensions are approximately $8000 \times 7000 \times 2000$ ft owing to the complex geometry. A uniform fracture aperture of 0.003 ft is assumed for the three fractures.

Figure 14: Frio field case with discrete fractures (shaded orange).

Table 3 lists the reservoir and fluid property information. A deliberate choice of isotropic and homogeneous permeability field is made in order to accentuate the presence of fractures. Fig. 15 shows the pressure and saturation profiles after 800 days. We make
two important observations: (1) the two longer fractures (along the reservoir length) are detrimental to recovery since the injected fluid shoots through and reduces sweep area, and (2) the shorter fracture (along the reservoir breadth) acts as a screen or shield to the fluid front and increases sweep area. Thus, well placement in a fractured reservoir requires additional considerations as opposed to reservoirs with no fractures.

![Pressure and Saturation Profiles](image)

**Figure 15:** Pressure (left) and saturation (right) profiles after 2.2 years.

**Conclusions**

We have modeled multiphase flows in fractured poro-elastic reservoirs where the fractures are modeled as surfaces (interfaces in two-dimensional reservoirs). The contrast between reservoir and fracture is fully resolved using different flow models and capillary pressure and relative permeability curves. A solution algorithm and numerical scheme based on MFMFE approximation has been described. The fracture geometry along with its non-planarity is accurately captured using general hexahedral elements. The model is validated against experimental lab data for spontaneous, capillary imbibition of a Berea sandstone core. Several numerical experiments, including a field case, have been performed which demonstrate that the recovery pattern is strongly influenced by the geometry and orientations of the fractures. These examples provide both qualitative and quantitative understanding of the underlying physical processes. The use of explicit flow models for both hydraulic and discrete fractures provide us with an accurate depiction of flow fields. This allows design and evaluation of hydraulic fracture jobs considering intricate details. Incorporating the geomechanical effects show that the influence of fractures on the stress field is more prominent around the production than the injection wells.
Nomenclature

\( \Gamma = \) fracture domain
\( \Omega = \) reservoir domain
\( \partial \Omega^{N,D} = \) reservoir flow boundary
\( \partial \Omega^{N,D}_o = \) reservoir mechanics boundary
\( \phi^* = \) porosity
\( \phi_o = \) reference porosity
\( \text{ref} = \) reference phase
\( \beta = \) oil (o) or water (w) phase
\( S_\beta = \) saturation of phase \( \beta \)
\( p_\beta = \) pressure of phase \( \beta \)
\( p_{c\beta} = \) capillary pressure of phase \( \beta \)
\( \rho_\beta = \) density of phase \( \beta \)
\( \rho_{\beta_0} = \) reference density of phase \( \beta \)
\( z_\beta = \) Darcy flux of phase \( \beta \)
\( z_t = \) total flux
\( \nu_\beta = \) viscosity of phase \( \beta \)
\( c_\beta = \) compressibility of phase \( \beta \)
\( k_\beta = \) relative permeability of phase \( \beta \)
\( q_\beta = \) source or sink term for phase \( \beta \)
\( q_{\beta o} = \) fracture leakage term for phase \( \beta \)
\( K = \) absolute permeability
\( g = \) acceleration due to gravity
\( w = \) fracture aperture
\( u = \) displacement
\( \sigma^{\text{por}} = \) Cauchy stress tensor
\( \varepsilon = \) strain tensor
\( \lambda, \mu = \) Lame parameters
\( \alpha = \) dimensionless Biot coefficient
\( M = \) Biot constant
\( f = \) body force
\( S_{wirr} = \) Irreducible water saturation
\( S_{or} = \) Residual oil saturation
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References


