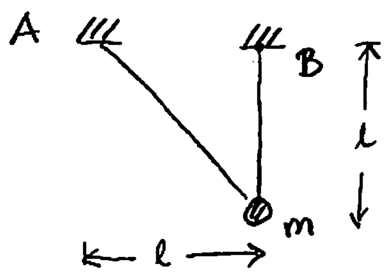
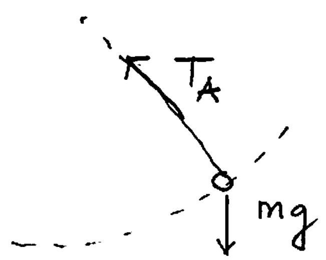
EM311M - Dynamics   Solutions
Exam 2
Wednesday, Mar 30, 2011, 6:00 - 8:00 p.m., WEL 1.308

1. The ball of mass m is suspended by cables A and B. Cable B is cut. Is the force in cable A going to increase or decrease ? Explain (5 points)



Dynamics :



$$m a_n = T_A - mg\cos 45^\circ$$

$$a_n = \frac{v^2}{\sqrt{2}\,l}$$

starts from rest $\Rightarrow v = 0$

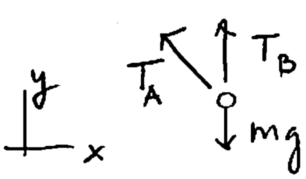
$\Rightarrow a_n = 0$

Statics :

$$\Sigma F_x = 0 \Rightarrow \underline{\underline{T_A = 0}}$$

$<$

$\therefore \quad \underline{\underline{T_A = mg\cos 45^\circ}}$



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2. Derive the principle of work and energy for a single particle. (5 points)

$$a_t\, ds = v\, dv \quad /\cdot m$$

$$\underbrace{m a_t}_{F_t}\, ds = m v\, dv$$

$$\underset{\parallel}{F_t\, ds} = m v\, dv$$

$$\underset{\sim}{F} \cdot d\underset{\sim}{r}$$

$$\int_A^B \underset{\sim}{F} \cdot d\underset{\sim}{r} = \int_{v_A}^{v_B} m v\, dv = \frac{m v^2}{2}\Big|_{v_A}^{v_B}$$

$$\therefore \quad \frac{m v_A^2}{2} + \int_A^B \underset{\sim}{F} \cdot d\underset{\sim}{r} = \frac{m v_B^2}{2}$$

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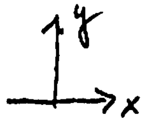
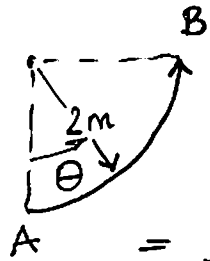
$$\boxed{T_A + U_{AB} = T_B}$$

3. Use the *definition of work* to compute the work of a constant force $\boldsymbol{F} = (-1, 3)$ [kN] along the circular segment AB (5 points)

Parametrization : $\begin{cases} x = 2\sin\theta \\ y = -2\cos\theta \end{cases}$





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$$\int_A^B \underset{\sim}{F}\cdot d\underset{\sim}{r} = \int_0^{\frac{\pi}{2}} \left(F_x \cdot \frac{dx}{d\theta} + F_y \frac{dy}{d\theta}\right) d\theta$$

$$= \int_0^{\frac{\pi}{2}} (-2\cos\theta + 6\sin\theta)\, d\theta = \left(-2\sin\theta - 6\cos\theta\right)\Big|_0^{\frac{\pi}{2}} = \underline{\underline{4}}$$

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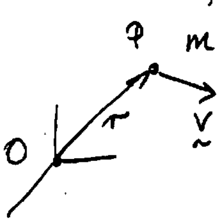
4. Derive the principle of angular impulse and momentum for a single particle. (5 points)

O - fixed



$$m\dot{\underset{\sim}{v}} = \underset{\sim}{F} \quad / \; \underset{\sim}{r}\times$$

$$(\underset{\sim}{r}\times m\dot{\underset{\sim}{v}}) = \underset{\sim}{r}\times\underset{\sim}{F}$$

$$(\underset{\sim}{r}\times m\underset{\sim}{v})^{\bullet} = \underset{\sim}{r}\times\underset{\sim}{F}$$

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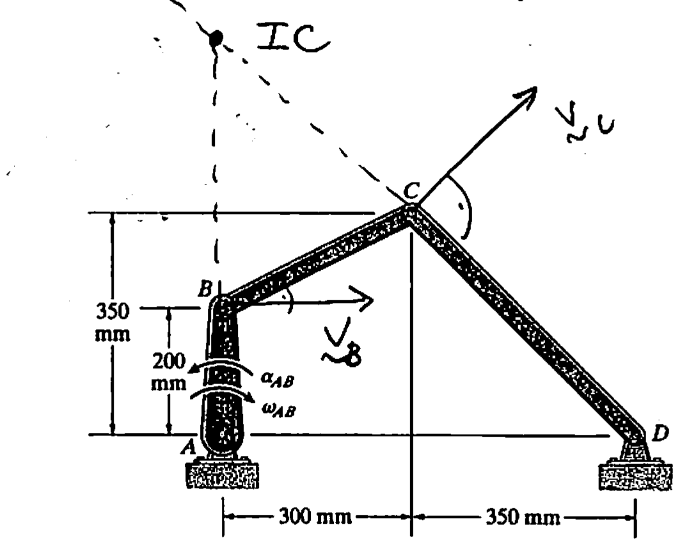
$$\boxed{\dot{\underset{\sim}{H}}_O = \underset{\sim}{M}_O}$$

moment about O

angular momentum wrt O

$$(\underset{\sim}{r}\times m\underset{\sim}{v})^{\bullet} = \underbrace{\cancel{\dot{\underset{\sim}{r}}\times m\underset{\sim}{v}}}_{\dot{r}=\underset{\sim}{v}}^{\,0} + \underset{\sim}{r}\times m\dot{\underset{\sim}{v}}$$

5. Identify the IC of zero velocity for the bar BC. (5 points)



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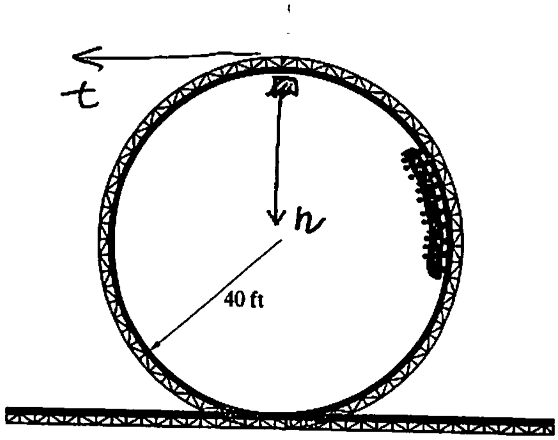
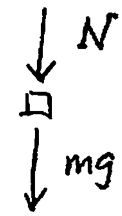
6. Suppose you are designing a roller-coaster track that will take cars through a vertical loop of 40-ft radius. If you decide that, for safety, the downward force exerted on a passenger by his or her seat at the top of the loop should be at least one-half the passenger's weight, what is the minimum safe velocity of the cars at the top of the loop? (25 points)



Free body diagram



N – force exerted by the seat on the passenger

Notice that we must have $N \geq 0$, if we do not want to rely exclusively on seat belts...

For safety, $N \geq \frac{1}{2} mg$

Eqn. of motion in the normal direction

$$\underbrace{m \frac{v^2}{r}}_{a_n} = N + mg$$ (10)

So $$m \frac{v^2}{r} - mg = \overbrace{N \geq \frac{mg}{2}}^{\text{the criterion!}}$$ (5)

$$\cancel{m} \frac{v^2}{r} \geq \frac{3}{2} \cancel{m} g$$

$$v^2 \geq \frac{3}{2} g r$$

$$v \geq \sqrt{\frac{3}{2} g r} = 43.95 \ \frac{ft}{s}$$

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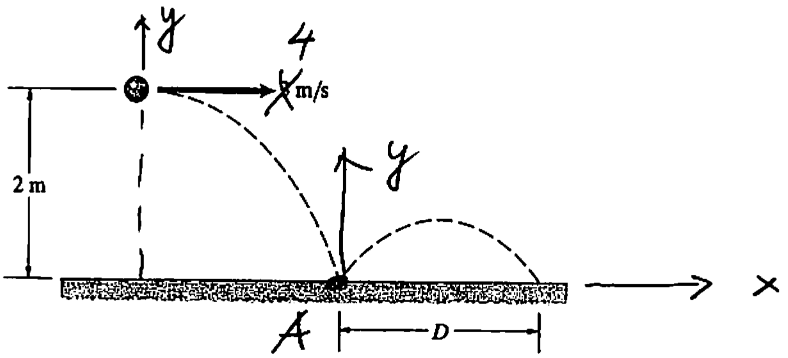
7. A ball is given a horizontal velocity of 4 m/s at 2 m above the smooth floor. Determine distance D between the ball's first and the second bounces if the coefficient of restitution is $e = 0.6$. (25 points)



Step 1: Motion before the first impact

$$\begin{cases} x = 4t \\ y = 2 - \dfrac{9.81t^2}{2} \end{cases} \qquad \dot{x} = 4 \qquad \dot{y} = -9.81t$$

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$y = 0 \Rightarrow t = 0.639\ [s]$

Step 2: Impact

Velocity of the ball before impact

$V_x^{before} = 4\ \frac{m}{s} \qquad V_y^{before} = -6.27\ \frac{m}{s}$

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Velocity of the ball after impact

$V_x^{after} = V_x^{before} = 4\ \frac{m}{s}$ (linear momentum in x is conserved!)

$e = \dfrac{V_y^{after}}{-V_y^{before}} \Rightarrow V_y^{after} = +3.76\ [\frac{m}{s}]$

Step 3: Motion between the impacts

Shifting coordinates to the place of first impact (A)

$$\begin{cases} x = 4t \\ y = 3.76t - \dfrac{9.81t^2}{2} = 0 \end{cases} \Rightarrow t = 0.767\ [s]$$

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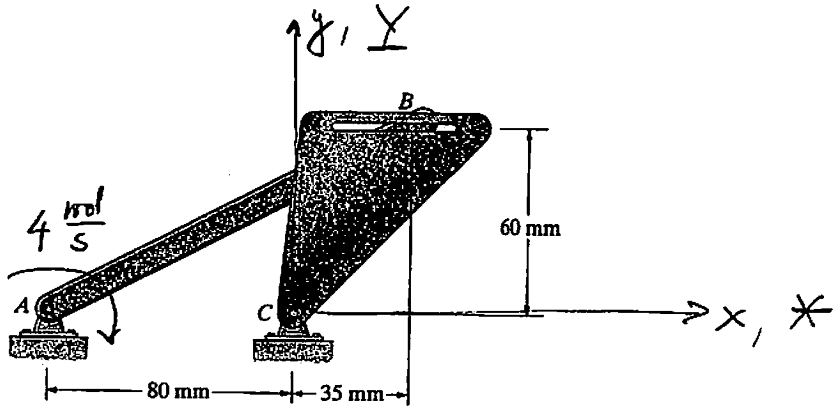
(new t, measured from the time of the first impact)

$\therefore \boxed{D = x = 3.07\ m}$

8. Bar AB has an angular velocity of 4 rad/s in the clockwise direction and an angular acceleration of 10 rad/s^2 in the counterclockwise direction. What is the acceleration of pin B relative to the slot ? (25 points)



Velocities :

$$\underset{\sim}{v}_B = \cancel{\underset{\sim}{v}_A}^{0} + \underset{\sim}{\omega}_{AB} \times \underset{\sim}{AB} = \begin{array}{r} \underset{\sim}{\omega}_{AB}\,(0,\,0,\,-4) \\ \times\ \underset{\sim}{AB}\,(0.115,\,0.06,\,0) \\ \hline (0.24,\,-0.46,\,0) \end{array} \quad \left[\frac{m}{s}\right]$$

$$\underset{\sim}{v}_B = \cancel{\underset{\sim}{v}_C}^{0} + \underset{\sim}{\omega}_{CB} \times \underset{\sim}{CB} + \underset{\sim}{v}_B^{rel} = \begin{array}{r} \underset{\sim}{\omega}_{CB}\,(0,\,0,\,\omega_{CB}) \\ \times\ \underset{\sim}{CB}\,(0.035,\,0.06,\,0) \\ \hline (-0.06\,\omega_{CB},\,0.035\,\omega_{CB},\,0) \end{array} + (v_{Bx}^{rel},\,0,\,0)$$

Comparing :

$$0.24 = -0.06\,\omega_{CB} + v_{Bx}^{rel}$$
$$-0.46 = 0.035\,\omega_{CB} \quad \Rightarrow \quad \omega_{CB} = -13.14\ \frac{rad}{s}$$

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$$\Rightarrow \quad v_{Bx}^{rel} = -0.549\ \frac{m}{s}$$

Accelerations :

$$\underset{\sim}{a}_B = \cancel{\underset{\sim}{a}_A}^{0} + \underset{\sim}{\alpha}_{AB} \times \underset{\sim}{AB} - \omega_{AB}^2\,\underset{\sim}{AB} = \begin{array}{r} \underset{\sim}{\alpha}_{AB}\,(0,\,0,\,10) \\ \times\ \underset{\sim}{AB}\,(0.115,\,0.06,\,0) \\ \hline (-0.6,\,1.15,\,0) \end{array} - 16\,(0.115,\,0.06,\,0)$$

$$= (-2.44,\,0.19,\,0) \left[\frac{m}{s^2}\right]$$

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$$\underset{\sim}{a}_B = \cancel{\underset{\sim}{a}_C}^{0} + \underset{\sim}{\alpha}_{CB} \times \underset{\sim}{CB} - \omega_{CB}^2\,\underset{\sim}{CB} + \underset{\sim}{a}_B^{rel} + 2\,\underset{\sim}{\omega}_{CB} \times \underset{\sim}{v}_B^{rel}$$

$$= \begin{array}{r} \underset{\sim}{\alpha}_{CB}\,(0,\,0,\,\alpha_{CB}) \\ \times\ \underset{\sim}{CB}\,(0.035,\,0.06,\,0) \\ \hline (-0.06\,\alpha_{CB},\,0.035\,\alpha_{CB},\,0) \end{array} - (13.14)^2 (0.035,\,0.06,\,0)$$

$$+ \ (a_{Bx}^{rel},\,0,\,0) + 2 \begin{array}{r} \underset{\sim}{\omega}_{CB}\,(0,\,0,\,\cancel{X}\,-13.14) \\ \times\ \underset{\sim}{v}_B^{rel}\,(1.028,\,0,\,0) \\ \hline (0,\,-13.5,\,0) \end{array}$$

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8) continued

Comparing accelerations:

$$-2.44 = -0.06\,\alpha_{CB} - 6.04 + a_{Bx}^{rel}$$

$$0.19 = 0.035\,\alpha_{CB} - 10.35 + 14.42$$

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$$\alpha_{CB} = -110.9 \ \frac{rad}{s^2}$$

$$a_{Bx}^{rel} = -3.05 \ \frac{m}{s^2}$$