EM386M/CAM386M FUNCTIONAL ANALYSIS IN THEORETICAL MECHANICS
List of Theorems to Be Covered on the Exams

Exam 1:

1. Principle of mathematical induction
2. Bijection between the family of all equivalence classes and the family of all partitions for a set
3. Properties of direct and inverse image
4. Characterization of a bijection
5. Comparability of cardinal numbers
6. Properties of open sets in $\mathbb{R}^n$
7. Properties of closed sets in $\mathbb{R}^n$
8. Relation between open and closed sets (duality principle)

Exam 2:

1. Properties of the interior operation
2. Properties of the closure operation
3. Characterization of accumulation (limit) points with sequences
4. Equivalence of continuity and sequential continuity in $\mathbb{R}^n$
5. The Bolzano-Weiestrass Theorem for Sets
6. The Bolzano-Weiestrass Theorem for Sequences

7. The Weierstrass Theorem
8. Characterization of $\liminf$
9. Characterization of a direct sum of two vector subspaces
10. Characterization of a Hamel basis
11. Existence of a Hamel basis in a vector space
12. Rank and Nullity Theorem
13. Characterization of a projection
14. Construction of a dual basis in a finite-dimensional space,
15. Properties of orthogonal complements
16. Properties of transpose operators
17. Relation between rank of a linear map and the rank of its transpose
18. Cauchy-Schwarz Inequality
19. Properties of adjoint operators

Exam 3:

1. Properties of a σ-algebra (Prop. 3.1.1)
2. Properties of an (abstract) measure (Prop. 3.1.6)
3. Properties of Borel sets (Prop. 3.1.4, 3.1.5 combined)
4. Characterization of Lebesgue measurable sets (Prop. 3.2.3, Thm 3.2.1)
5. Cartesian product of Lebesgue measurable sets (Thm. 3.2.2)
6. Properties of measurable (Borel) functions (Prop. 3.4.1)
7. Properties of Lebesgue integral (Prop. 3.5.1)

8. Fatou's Lemma
9. Lebesgue Dominated Convergence Theorem (for non-negative functions, Thm. 3.5.2)
10. Hölder and Minkowski inequalities,
11. Properties of open sets, properties of closed sets, properties of the operations of interior and closure (all in context of general topological spaces),
12. Characterization of open and closed sets in a topological subspace.
13. Characterization of (globally) continuous functions (Prop. 4.3.2),
14. Properties of compact sets.
15. The Heine-Borel Theorem,
16. The Weierstrass Theorem,

Additional material for the final:

1. Properties of sequentially compact sets (Prop. 4.4.5),
2. Hölder and Minkowski inequalities for sequences,
3. Completness of Chebyshev, l^p, and L^p spaces,
4. Bolzano-Weiestrass Theorem,
5. Banach Contractive Map Theorem.